A policy for written calculations

devised by

teachers in the Hexham Partnership of Schools

30/11/10

This guidance has been developed by teachers in the Hexham Partnership of Schools in response to:

- concerns from teachers about consistency of approaches to teaching written calculations;
- requests from parents for information so that they can help their children at home in ways which complement methods taught at school.

Principal contributors have been teachers from Corbridge Middle and First Schools, with other Partnership schools providing additional ideas.

The policy is intended to be a working document. It is not meant to be dictatorial: it is for Partnership schools to use as they see fit. It may be that revisions will be required in the future. Please send any comments on the policy to Peter.Woodward@northumberland.gov.uk so that the views of users may be incorporated in future versions.

Why have a policy for written calculation strategies?

Some principles which underpin the philosophy of the National Numeracy Strategy of 1999 and its later revisions are that:

- Good mental calculation skills are an essential complement to written strategies.
- Children need to develop a range of strategies for calculation, and need to know which to use in a given situation.
- at all stages of learning, children should be given opportunities to *understand* the methods they encounter; i.e. they should not be taught meaningless "tricks".
- Children should learn a progression of strategies in each of the "four operations", leading eventually to the most efficient methods.

The complementary nature of addition and subtraction, and of multiplication and division, should be emphasised at every opportunity.

These are all sensible principles. However, problems can arise if….

- we attempt to teach a method in isolation, without reference to those which precede and those which follow.
- children are exposed to too many methods and end up confused.
- we are too rigid in our approach, and only teach a particular method at a particular time, instead of teaching something when children are ready.

Mental or written?

A lot of everyday maths is performed mentally, so children should be encouraged to use mental skills when they are confident that they can do so accurately. However, children can be over-confident about their mental abilities, or perhaps would just prefer to write less, and so may opt for a mental method when they should not. If a written method is being taught, it is perfectly reasonable to insist that it is used during tasks designed to practise it. However, in individual problem solving situations, part of the process of tackling the problem will involve deciding what kind of calculation method to use, and whether this should be mental or written. Each and every time a child looks at a problem involving calculation they should go through these questions:

- Can I tackle this calculation mentally?
- If not, which is the most efficient written method I know to solve it?
- Do I have a rough idea of what the answer will be?

This is the area where we need to step back and allow some independence. In this type of situation we cannot be the judge for each and every child as to what is the most suitable method. They must choose for themselves. However, progression in the teaching of calculation methods should lead children as far as possible down the road towards the most efficient method, i.e. one which is quick and accurate (accuracy being more important than speed).

Other principles:
1) Lists of vocabulary on classroom walls which begin “These words mean add…” etc. are not necessarily a good idea. For example, if an “Addition list” contained the phrase “more than”, this would be misleading, since the problem: “How many more than 34 is 57?” is not one that involves addition. Lists which quote words and phrases out of context are not helpful.

2) Problem solving is not something to “tack on” to the end of work on calculation. Neither is it just calculation with some kind of units of measurement built in. Calculations should be given a context at every opportunity. Children can visualise what is going on much more easily if they know they are dealing with pence, marbles, sweets, people etc.

Calculation with decimal numbers presents additional difficulties if the numbers are abstract. What does the number 4.8 mean to a child? In isolation, probably very little. Decimal numbers are often made more real to children, and therefore more understandable, if presented in the context of money. Many children who would struggle with the calculation
8.4 ÷ 5 would have a lot less difficulty with £8.40 ÷ 5. Contextualising such calculations need not be confined to money, however. If we are concentrating on numbers with one decimal place, money is not an appropriate context. However, measurement in centimetres (to the nearest millimetre) is ideal. The length 4.6 cm has meaning for children which the abstract number 4.6 does not.

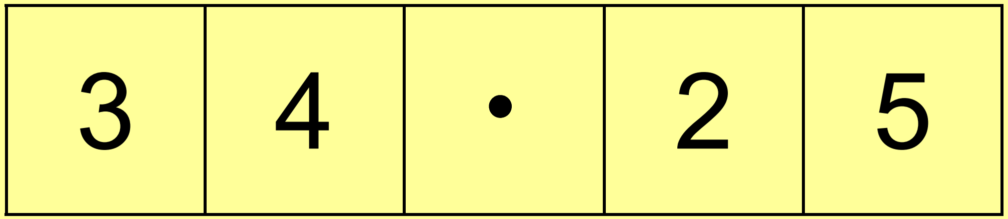
Contents

Page

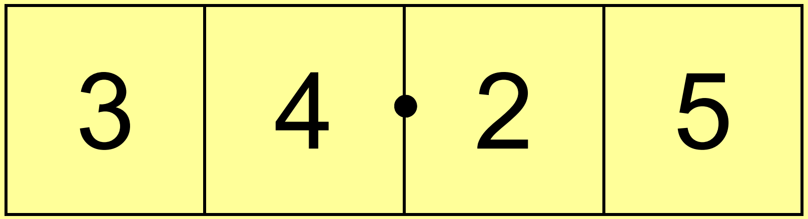
Notes on writing numbers

Assuming that we use squared paper to write numbers and calculations, it is an obvious advantage to stick to the rule of **one digit per square**. When writing decimals it is incorrect to assign the decimal point a square of its own. The decimal point represents a dividing line between units and tenths, and therefore should be written on the line between the units and tenths squares.

INCORRECT



CORRECT

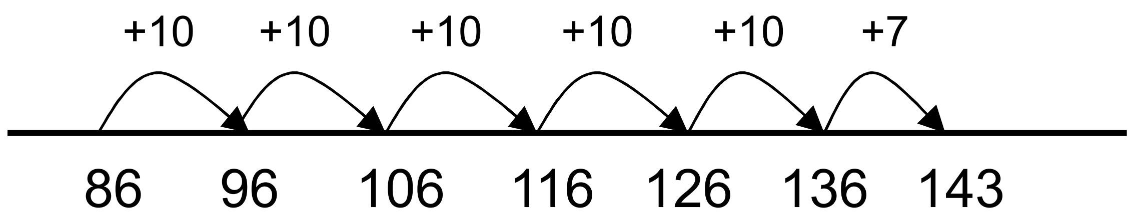


Adhering to this principle helps when introducing methods for multiplying and dividing by 10, 100 and 1000, since there is less confusion about how many places to move digits

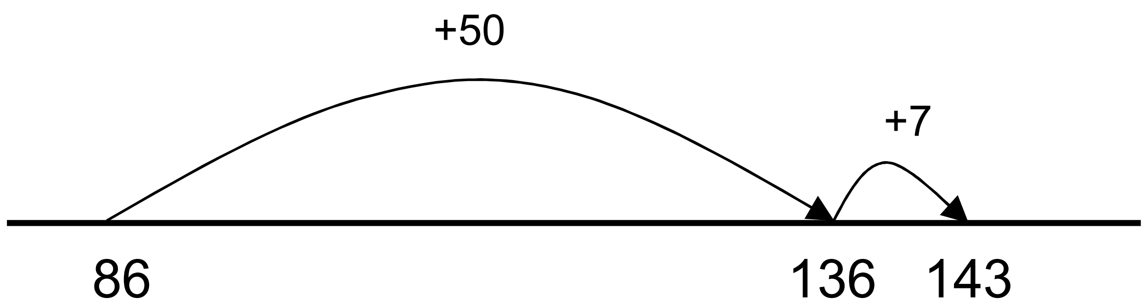
ADDITION 1

Counting on using a number line, in tens then in units

86 + 57



leading to



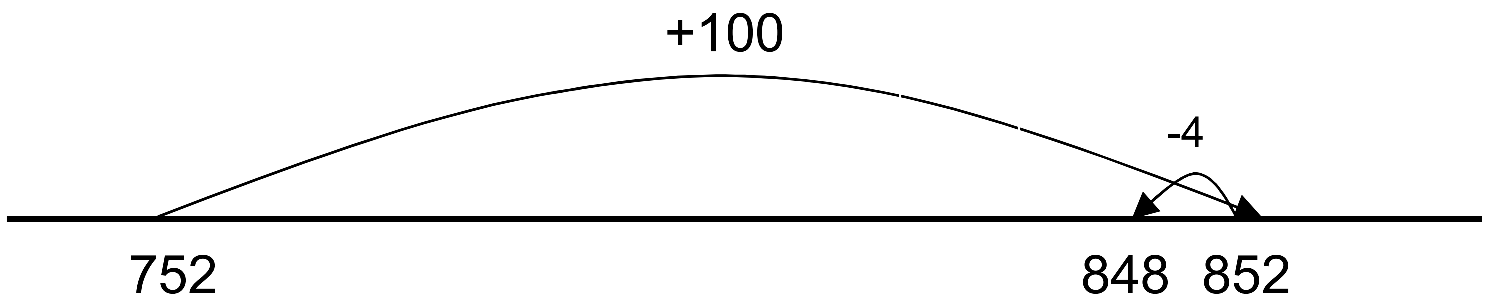
What to say:
Start at 86 (the larger number) on the number line. Count on 50 in tens first and then 7 units.
or
Count on 50 in one jump, then 7 units.

ADDITION 2

Counting on using a number line, adding too much, then compensating

752 + 96

What to say:
Count on 100, then take away (subtract) 4.



Addition on a number line becomes inefficient once the numbers involved become of the order of HTU + HTU. Once these sizes of numbers are reached other more efficient methods should be used.
NB. Many children will be able to use a mental method for this question: recognising that 96 is near 100, adding on 100, then subtracting 4. This is appropriate for this particular calculation, but will not be so for most others.

ADDITION 3

Partitioning

86 + 57

80 + 50 = 130
6 + 7 = 13
143

675 + 148

600 + 100 = 700
70 + 40 = 110
5 + 8 = 13
823

What to say:

Writing down each sub-total helps you to remember what has been done and what still needs to be done. Be careful to place sub-totals correctly underneath one another.

Adding most significant digits first reinforces number line methods. Children may "see" a way to do the second calculation mentally, by adding on 150 and subtracting 2. This is entirely appropriate for this particular calculation, but will not be the case for most others.

ADDITION 4

Expanded vertical method

What to say:
Add hundreds first, then tens, then units.

What to say:
Add units first, then tens, then hundreds.

387		387
+475		+475
700	then →	12
150		150
+ 12		+700
862		862

Adding most significant digits first reinforces number line methods. Adding least significant digits first prepares the way for the compact vertical (standard) method.

When using this method, always refer to the actual value of the digits concerned, e.g. 8 tens + 7 tens = 15 tens. It may be tempting to say more simply 80 + 70 = 150, but the former way actually paves the way better for the compact vertical method (see next page).

ADDITION 5

Compact vertical (standard) method

What to say to begin with:

A) 7 + 5 = 12. Put 2 in the units column and carry the 1 ten to the tens column.

B) 8 tens + 7 tens + 1 ten = 16 tens.
Put 6 tens in the tens column carry 1 hundred to the hundreds column.

C) 3 hundreds + 4 hundreds + 1 hundred = 8 hundreds.

```
  387
 +475
 ----
  862
 ----
  11
```

What to say eventually:

A) 7 + 5 = 12, which makes two of these and one of these

and so on. The phrase "of these" can be used in any column, simplifying the language needed.

What not to say:

7 + 5 = 12. Put 2 down and 1 on the doorstep.

The carried ten can be placed as shown or above the top line next to the 7, whichever children feel most comfortable with.

ADDITION 6

Compact vertical method with decimal numbers

45.3 + 0.74 + 156

```
  45.30
   0.74
+156.00
 202.04
 111
```

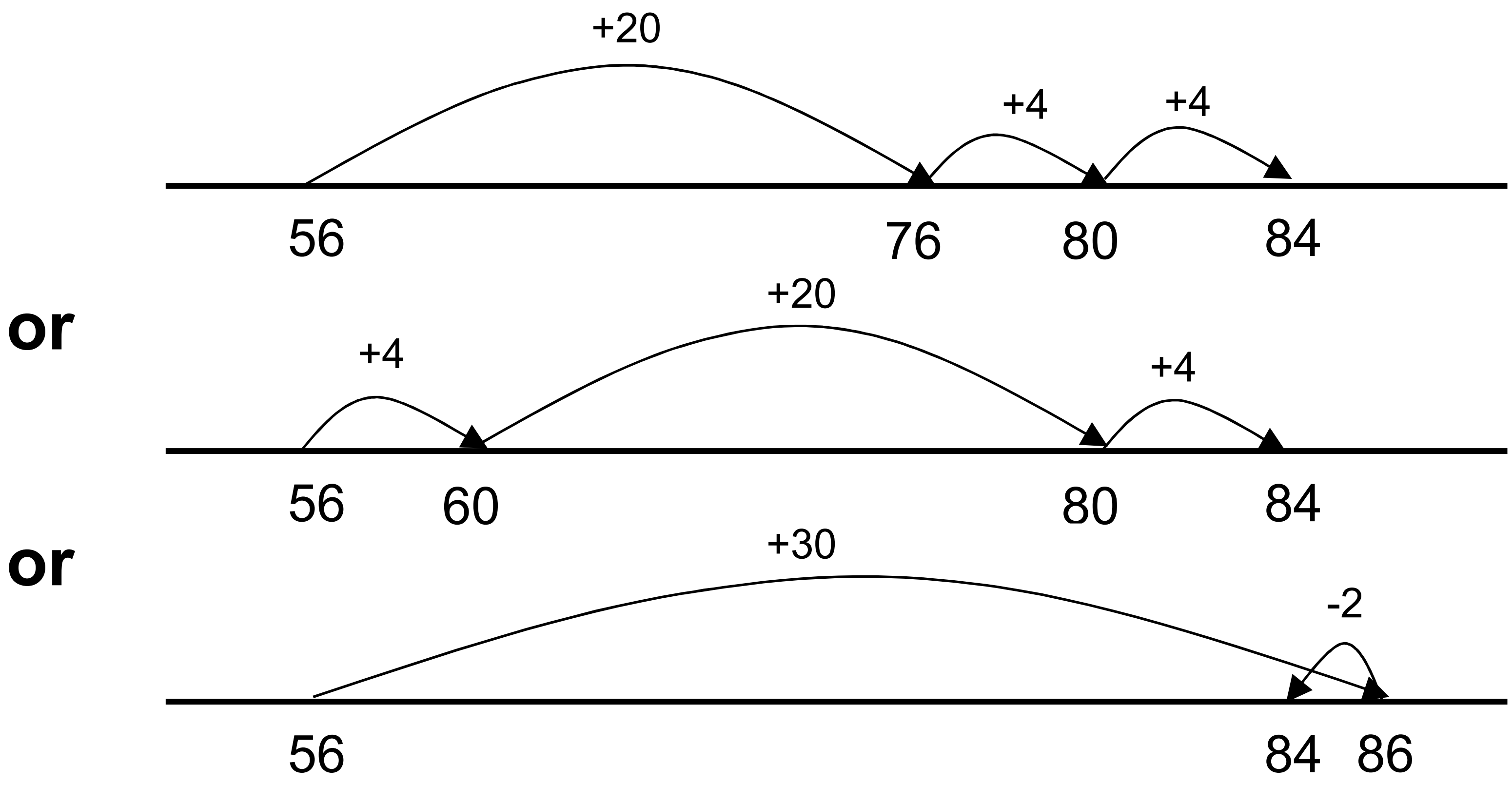
Problems with this type of calculation tend to stem from how the numbers are set out in the first place. Stress that the units must be placed underneath one another (likewise tens, hundreds etc.) Stressing that decimal points are part of the column system itself and must also be placed in line also helps. They should be placed on the dividing line between units and tenths, not in a space of their own! Filling empty spaces after the decimal point with zeros helps the correct alignment of numbers. If a number has no decimal point, then encourage rewriting it with one, plus any place-holder zeros. This also helps correct alignment.
The phrase "of these" (see previous page) comes in handy to simplify language when adding any column of digits.

SUBTRACTION 1

Counting on using a number line.

84 - 56

What to say:
Count on from 56 until you get to 84. Find out how much you've moved.



84 - 56 = 28

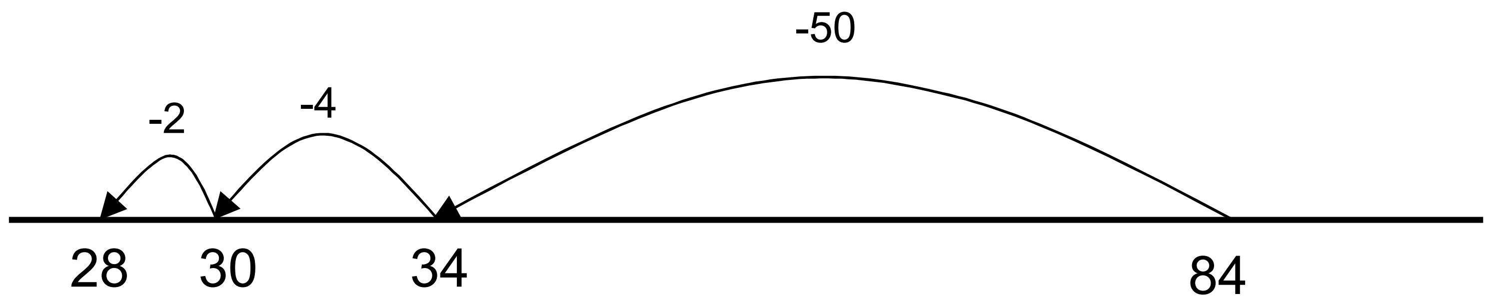
This is an especially appropriate method when the numbers are close together. It is very useful if a problem is presented to children as: ***"Find the difference between 56 and 84."*** It is also the method that is likely to occur to children if a problem is encountered which uses language like: ***"How much must be added to 56 to make 84?"*** or ***"How much more than 56 is 84?"***.

SUBTRACTION 2

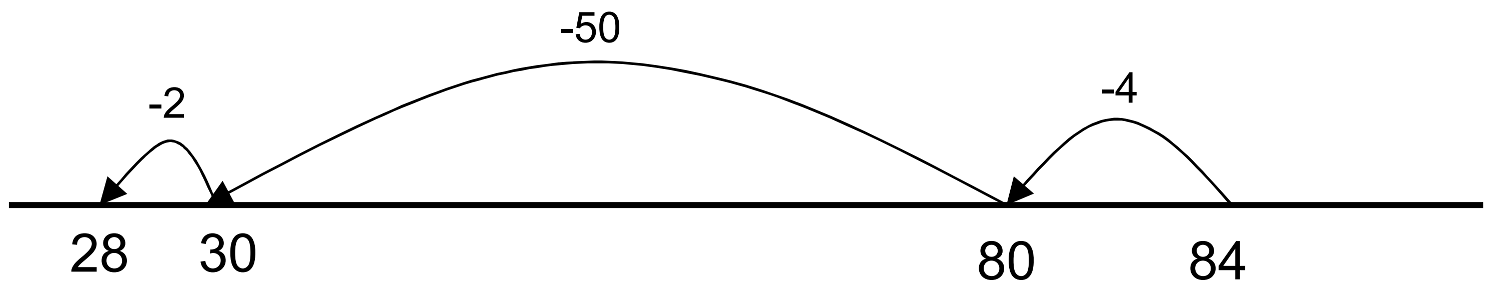
Counting back using a number line.

84 - 56

What to say:
Start at 84 and count back 56. Find where you reach.



or



84 - 56 = 28

This is the method that is likely to occur to children if a problem is encountered which uses language like: ***"What is 84 take away (subtract) 56?"*** or ***"What is 56 less than 84?"*** This number line may also be presented vertically, a helpful idea when children encounter work on change in temperature, height etc.

SUBTRACTION 3

Expanded decomposition

154 - 36

	100	+	50	+	4
-			30	+	6

↓

			40		10
	100	+	~~50~~	+	4
-			30	+	6

↓

			40		10		
	100	+	~~50~~	+	4		
-			30	+	6		
	100	+	10	+	8	=	118

What to say:

A) Split both numbers into their hundreds, tens and units.

B) Start with the units. 4 take away 6* isn't possible, so change+ a ten into 10 units, making 14 altogether.

C) 14 take away 6 = 8
40 take away 30 = 10
100 take away 0 = 100

D) Add the parts back together.
100 + 10 + 8 = 118

What not to say:

***6 from 4 (correct but confusing)**

+*borrow* one or *borrow* a ten (there is no borrowing going on!)

This method could be omitted altogether for children who have a very good understanding of place value and do not need the visual support it provides.

SUBTRACTION 4

Standard decomposition

What to say to start with:
A) Start with the units. 2 take away 6* isn't possible, so change⁺ a ten into 10 units, making 12 altogether.
C) 12 take away 6 = 6
4 tens take away 3 tens = 1 ten
1 hundred take away 0 = 1 hundred

What not to say:

- ***6 from 2**
- **⁺borrow one or borrow a ten**
- **2 take away 6 can't be done so cross out the 5, put a 4 and put the 1 next to the 2.**
- **40 take away 30 (It *is* 40 take away 30, but it's more helpful for later if you say 4 tens instead of 40 at this point.)**

```
  1 5 2
-   3 6
  -----
    ↓
    4
  1 5̸ ¹2
-   3 6
  -----
    ↓
    4
  1 5̸ ¹2
-   3 6
  -----
  1 1 6
  -----
```

What to say eventually:
A) Start with the units. 2 take away 6* isn't possible, so change⁺ a ten into 10 units, making 12 altogether.
C) 12 take away 6 = 6
4 *of these* take away 3 *of these* = 1*of these*.
1 *of these* take away 0 = 1 *of these*

What not to say:
As in the left hand box

This layout does not mean that the calculation is written out 3 times! It is shown in this way just to make clear the sequence of operations.

The most efficient method for subtraction. We should aim for as many children as possible being competent with this method. When teaching vertical subtraction, it is worth stressing to children that there are not two separate numbers present (as in addition). This is a common misconception. The only number present is the upper number; from which the lower must be taken. Other common errors include: (a) 2 - 6 = 4 (b) attempting too much mentally. However, children should not be made to use a written method all the time. The calculation 200 - 147 is quite difficult to do by standard decomposition, but is relatively easy to do mentally.

SUBTRACTION 5

Decomposition with decimal numbers

15.4 - 6.78

```
  1 5 . 4
-   2 . 3 8
-----------
      ↓

  1 5 . 4 0
-   2 . 3 8
-----------
      ↓
        3
  1 5 . 4 ¹0
-   2 . 3 8
-----------
  1 3 . 0 2
-----------
```

What to say:

A) Make sure digits with equal value (and decimal points) are in line vertically

B) Add a place-holder zero to 15.4

C) Start with the units. 0 take away 8 isn't possible, so change ten of these into one of these.

D) 10 of these take away 8 of these = 2 of these
3 of these take away 3 of these = 0 of these
5 of these take away 2 of these = 3 of these
1 of these take away 0 = 1 of these

What not to say:

Don't call all digits by their true value (3 tenths, 8 hundredths etc.) It's too confusing.

Competence with the decomposition method pays dividends when dealing with decimal numbers, when other methods become cumbersome and confusing. However, once again, children should not be made to use a written method all the time. The calculation 20 - 12.95 is difficult to do by standard decomposition, but is relatively easy to do mentally. Furthermore, it is an important life skill to be able to calculate mentally the change from a £20 note if you spend £12.95.

MULTIPLICATION 1x

Multiplying whole numbers by 10

62 x 10

H	T	U
	6	2
6	2	0

compulsory

What to say:
The value of each digit becomes 10 times bigger, so units become tens and tens become hundreds.
Move all digits one place to the left into columns which are 10 times bigger.
Fill in the space in the units with a place-holder zero.

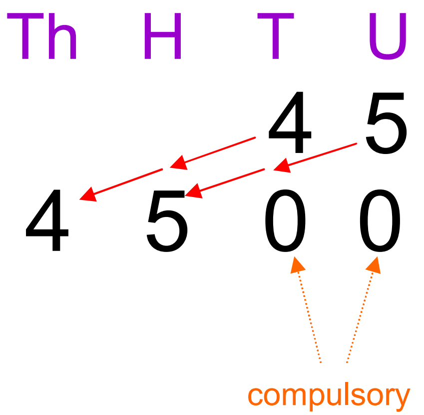
What not to say:
To multiply by 10 just add a zero.

Writing the answer underneath the original number as shown helps to stress the movement of the digits.
Saying to children that you can "just add a zero" creates problems later when multiplying decimal numbers by ten.   3.2 x 10 is not 3.20 !

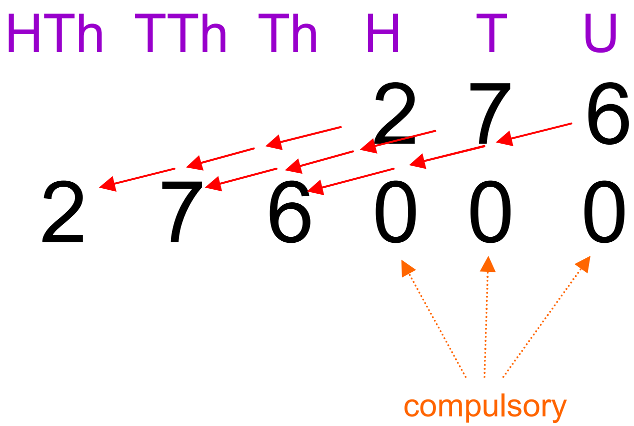
MULTIPLICATION 2x

Multiplying whole numbers by 100 and 1000

45 x 100



276 x 1000



What to say:
Slide all the digits 2(or 3) places to the left. Fill empty columns with 2 (or 3) place-holder zeros.

What not to say:
To multiply by 100 (or 1000) just add 2 (or 3) zeros.

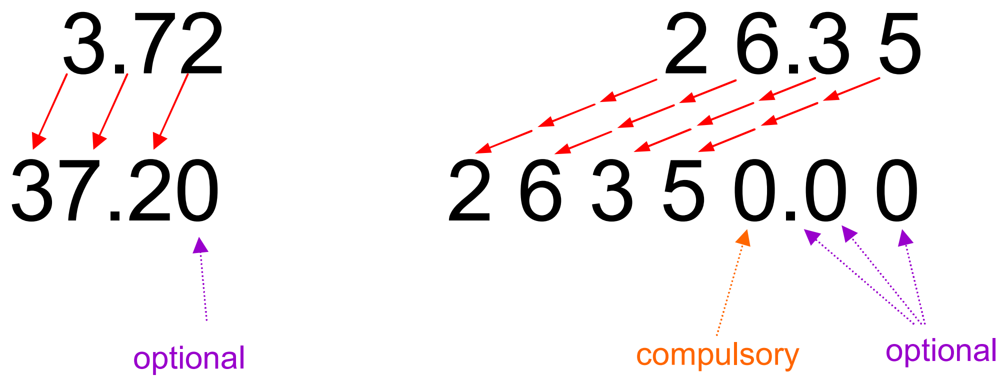
Writing the answer underneath the original number as shown helps to stress the movement of the digits.
It is not compulsory to draw in the arrows, but it may help initially. They may be drawn as a series of "hops" to emphasise how many places the digits are moving.
Stress to children that, once one of the digits has been placed correctly, others slot into place in the same order.

MULTIPLICATION 3x

Multiplying decimal numbers by 10, 100 and 1000.

3.72 x 10

26.35 x 1000



What to say:
As with whole numbers, slide all digits 1 (2, 3) spaces to the left. Fill in any spaces before the decimal point with place-holder zeros.

What not to say:
Slide the decimal point 1, 2, 3 spaces to the right.

It is important to emphasise that the decimal point is part of the place value system and therefore does not move.
Writing the answer underneath the original number as shown helps to stress the movement of the digits.

DIVISION 1x

Dividing whole numbers by 10

270 ÷ 10

H T U

2 7 0

2 7. 0

optional

346 ÷ 10

H T U. t

3 4 6

3 4. 6

compulsory!

What to say:

The value of each digit becomes 10 times smaller, so hundreds become tens and tens become units.

Move all digits one place to the right into columns which are 10 times smaller.

What not to say:

To divide by 10 just knock off a zero.

Writing the answer underneath the original number as shown helps to stress the movement of the digits.

Saying to children that you can "just knock off a zero" is of no use if the units digit isn't a zero!

Encouraging children to modify 346 into 346.0 before dividing can help to establish the position of the decimal point before any movement of digits is undertaken.

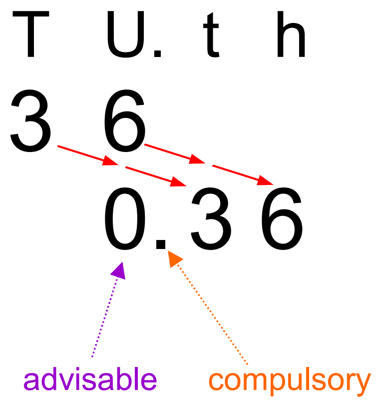
DIVISION 2x

Dividing whole numbers or decimals by 100 and 1000

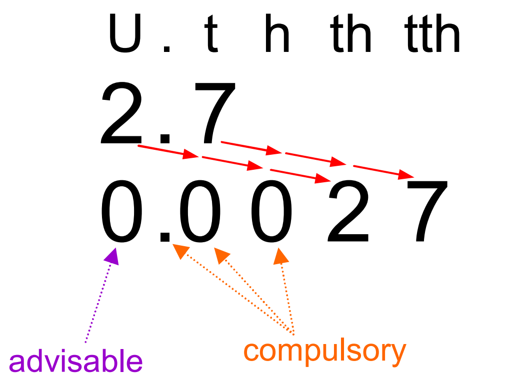
What to say:
Slide all the digits 2(or 3) places to the right. Fill relevant empty columns with place-holder zeros.

What not to say:
To divide by 100 (or 1000) knock off 2 (or 3) zeros.

$36 \div 100$



$2.7 \div 1000$



Writing the answer underneath the original number as shown helps to stress the movement of the digits.

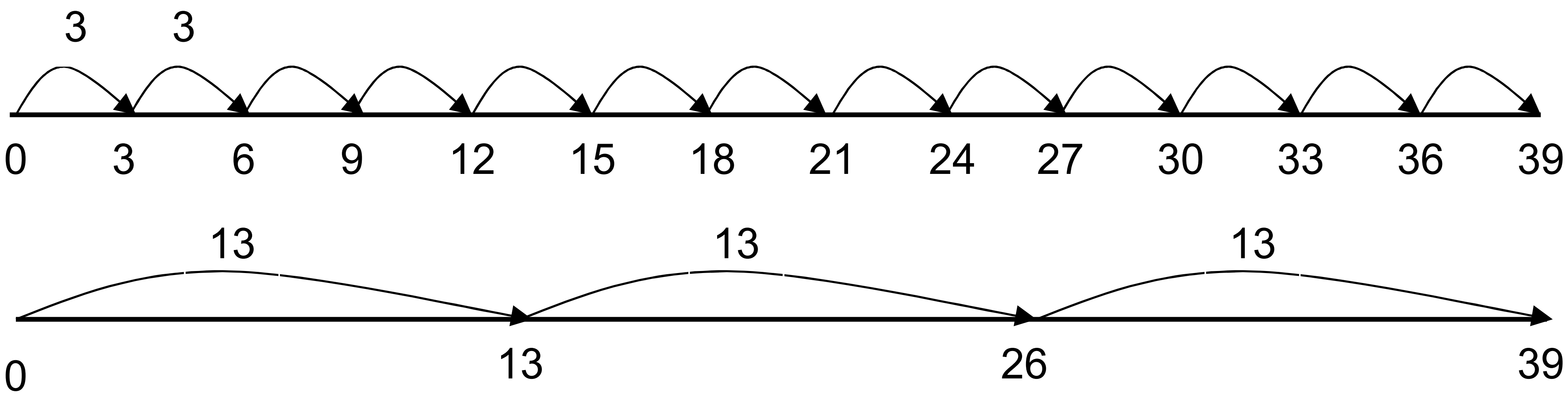
Saying to children that you can "just knock off zeros" is of no use if the relevant digits aren't zeros!

MULTIPLICATION 1

Repeated addition on a number line

What to say:	What not to say:
13 threes	13 times by 3
13 lots of 3	13 timesed by 3
13 times 3	
13 multiplied by 3	

13 x 3



The sign "x" and the word "times" are ambiguous. In common usage the sign can represent "lots of", "times", "multiplied by" etc. This creates a problem, since "13 lots of 3" is represented by the top number line and "13 multiplied by 3" by the bottom one. "13 times 3" could possibly be either. It is so well embedded in everyday language (with either meaning) that it is pointless trying to insist that it has one or the other. However, if work on written methods is preceded by lots of reinforcement of the idea that a x b gives the same answer as b x a, then pupils should already be in the habit of assessing a multiplication calculation to see which way round it could be tackled most easily.

MULTIPLICATION 2

Partitioning then multiplying

42 x 3

= (40 x 3) + (2 x 3)
= 120 + 6 = <u>126</u>

or **40 x 3 = 120**
2 x 3 = <u>6</u>
<u>126</u>

What to say:
Split 42 into 40 and 2.
Multiply 40 by 3 and then 2 by 3 and add the answers together.

It is assumed that the various ways of how to tackle the calculation 40 x 3 have been taught as mental strategies. Counting up in threes 40 times is obviously not efficient! Counting 3 lots of 40 is relatively straightforward. Alternatively, 40 may be considered as
4 x 10 and the calculation rearranged:

40 x 3 → 4 x 10 x 3 → 4 x 3 x 10 → 12 x 10 → 120

When teaching written methods, it is best to limit the choice of numbers used to multiplication tables which children actually know.
Although this is an appropriate method for multiplying two digit numbers and larger by a single digit, is not appropriate for multiplying by a two digit number. If this type of setting out is used for 42 x 13 it is a common error to assume that all that need to be done is (40 x 10) and (2 x 3). Grid multiplication helps overcome this problem (see Multiplication 3).

MULTIPLICATION 3

Grid method

67 x 4

	60	7
4	240	28

240 + 28 = 268

36 x 13

	30	6
10	300	60
3	90	18

=

300 + 60 + 90 + 18 = 468

What to say:

Tips: The top left hand cell will always be the largest of the answers in the table. The bottom right answer will be the smallest, and will be the only one which can have any units.

The second example shows the advantage of this method over the previous one: The grid itself acts as a reminder that all four "part-products" must be found.

Before attempting two-digit x two-digit multiplication pupils must be very secure with mentally calculating 30 x 10, 40 x 20, 60 x 30 etc.

This method *can* be used for multiplying larger (i.e. 3- and 4-digit) numbers, but it is really too cumbersome for this.

It can also be used to multiply decimal numbers, e.g. 3.6 x 5, but see notes on Multiplication 7.

MULTIPLICATION 4

Vertical method - expanded

23 x 8

```
   2 3
 x   8
 -----
   2 4   (8 x 3)
 1 6 0   (8 x 20)
 -----
 1 8 4
 -----
```

46 x 32

```
     4 6
   x 3 2
   -----
     1 2   (2 x 6)
     8 0   (2 x 40)
   1 8 0   (30 x 6)
 1 2 0 0   (30 x 40)
 -------
 1 4 7 2
 -------
```

What to say:
Make sure that every part of the multiplication is done.
Check that the biggest and smallest of the answers are where you expect them to be.
Check that your list of answers to be added are aligned correctly.

The separate sub-calculations can be done in any order, but it is better to start with units, since this provides a better foundation for the compact method.

The calculations in brackets need not be written unless teacher and/or pupil decides that it would be helpful to do so. In any event, doing so should be a temporary measure, omitted when confidence with the method is achieved.

An advantage of this method over the grid method is that "part-products" are automatically arranged so that they are easy to add together. A disadvantage is that there is no *visual* check on whether all "part-products" have been calculated. It relies on the pupil knowing how many there should be for a given calculation.

MULTIPLICATION 5

Vertical compact method (single digit)

23 x 8

```
    3  2
x      7
---------
 2  2  4
---------
 2  1
```

What to say initially:

Seven twos = 14. Put 4 in the units and carry 1 ten forward.
Seven lots of 3 tens = 21 tens. Add the ten carried forward, making 22 tens.
22 tens = 2 hundreds and 2 tens.

What to say eventually:

Seven twos = 14. That's 4 of these and 1 of these.
Seven lots of three of these = 21 of these, and so on.

What not to say

7x 2 = 14. Put 4 in the units and carry 1.
7 x 3 = 21; add the 1, making 22.

The phrase "of these" can be applied to any column.

MULTIPLICATION 6

Vertical compact method (two digit)

46 x 32

```
   4 6
 x 3 2
 -----
   9 2
 1 3 8 0
 -------
 1 4 7 2
 -------
```

What to say:
Multiply 46 by 2 first.
Then multiply 46 by 30. Consider this as 46 x 10 x 3. Begin by placing a zero in the units column, which means that everything in this answer will be moved one place to the left and so multiplied by 10. Then multiply 46 by 3.
Add the two answers together.

What not to say
When multiplying by 30, put a zero down and multiply by 3.

This is the most efficient way of multiplying large whole numbers. However, it should not be attempted without a thorough grounding in other methods.

Be very careful where carrying figures are written. There may be carrying figures on three different lines of the whole calculation!

MULTIPLICATION 7

Multiplication with decimals

6.7 x 4

↓ x10

67 x 4

```
   6 7
 x   4
 2 6 8
```

268 —÷10→ 26.8

3.5 x 1.2

↓ x10 ↓ x10

35 x 12

```
     3 5
   x 1 2
     7 0
   3 5 0
   4 2 0
```

420 —÷100→ 4.20

What to say:

Multiply the decimal number(s) by 10, 100 etc. until they become whole numbers.
Then do the main calculation.
Then divide the answer by 10, 100 etc. as appropriate to find the answer to the original calculation.
Alternatively, you could refer to "scaling up" the original number(s) and "scaling down" the answer to the calculation.

What not to say:

Count the number of decimal places in the question and put this many in the answer.

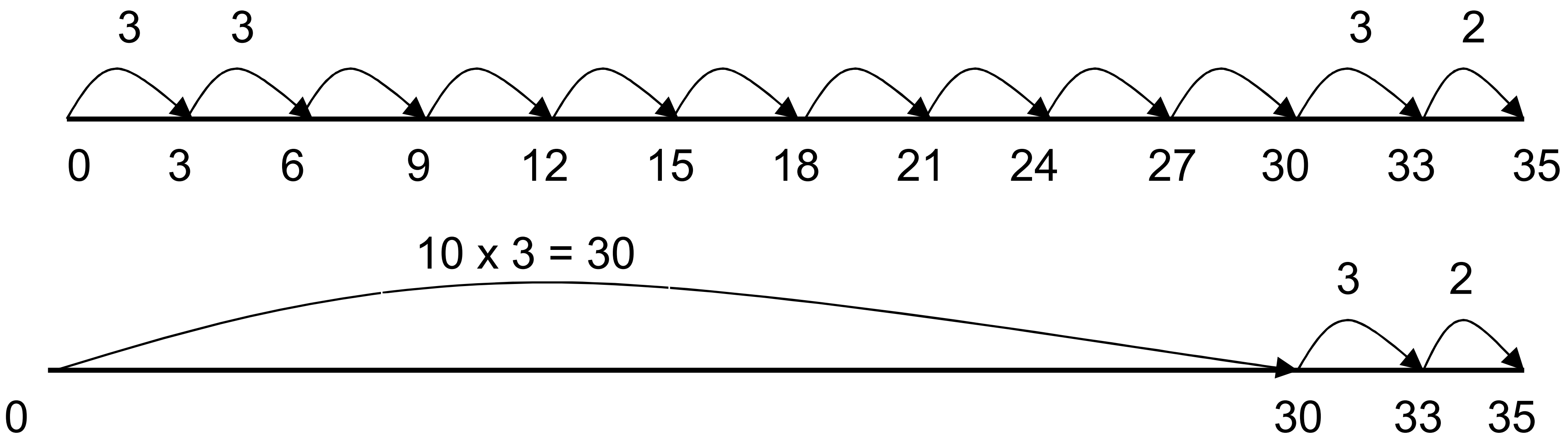
Multiplication with decimal numbers should not be introduced only when all other methods have been taught. It should be introduced around the time when grid multiplication is first taught (See Multiplication 3). Whatever method is in use, the principle should be the same. i.e. *do not* attempt to teach actual multiplication using decimal parts (e.g. 6 x 0.12 or 0.06 x 0.9). *Always* teach pupils to scale up the original numbers and scale down the answer.

DIVISION 1

Repeated addition on a number line (division by grouping)

35 ÷ 3

What to say:
Count up in threes as far as you can
Or, if possible, add 10 groups of 3 at a time.



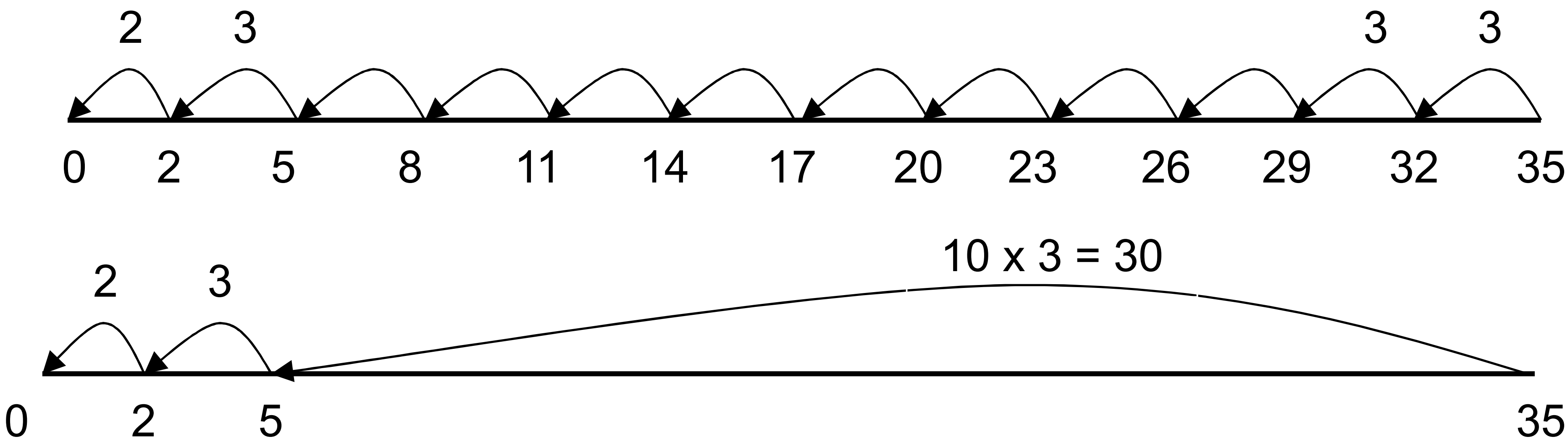
This method is an example of division by **grouping**, in this case arranging a number of objects in **groups of 3.** Note that this is quite distinct from division by **sharing,** where the number of objects is **shared between 3** (people). Confusingly, both are called division, but they are in reality two entirely different physical processes. It is not even correct to say that they have the same answer, since 24 ÷ 4 may mean grouping 24 into 6 groups of 4, or sharing 24 into 4 sets of 6. It is important to remember that the words "divide" and "division", plus the symbol "÷" are ambiguous. Phrases such as "group in fours" or "share between 5 people" have a single meaning, but "divide by 7" can mean different things.
Note that, when sharing, we should use the phrase "share between", not "share by", which has no practical meaning.

DIVISION 2

Repeated subtraction on a number line (division by grouping)

What to say:
Take away groups of 3 as far as you can.
Or, if possible, take away 10 groups of 3 at a time.

35 ÷ 3



This method is also an example of division by **grouping**. It is often more difficult for children to handle since counting down is generally harder than counting up, and may start from a number which is not a multiple of the divisor. However, it should be covered with children because the practical solution to "How many threes in 35?" would involve starting with 35 objects and repeatedly removing groups of three, not starting with zero and adding threes. If multiplication is repeated addition, then division (by grouping) is repeated subtraction.

DIVISION 3

"Short" division by sharing

What to say to begin with:
Share 7 tens between 3 people.
How many will they get each?
Two tens each with 1 ten left over.
Change the 1 ten left over into 10 units, making 14 units altogether.
Share 14 units between 3 people.
How many will they get each?
Four each, with 2 left over (remainder).

74 ÷ 3

$$\begin{array}{r} 2\ \ 4\ \ r\,2 \\ 3\,\overline{)\,7\ \ {}^{1}4} \end{array}$$

What to say eventually:
Share 7 of these between 3 people. They will get 2 of these each with 1 left over.
One of these will make 10 of these …… etc.

What not to say:
How many threes in 7?
How many threes in 70?
Three goes into 7 twice remainder 1, etc.

Note that, when sharing, we should use the phrase "share between", not "share by", which has no practical meaning.

DIVISION 4

"Short" division by sharing with decimal numbers

4.71 ÷ 3

```
   1 . 5  7
  ─────────
3 ) 4 . ¹7 ²1
```

What to say:

Share 4 of these between 3 people. They will get 1 of these each with 1 left over.
One of these will make 10 of these …… etc.

What not to say:

How many threes in 4?
Three goes into 4 once remainder 1, etc.

Using the phrase "of these" reinforces that each digit stands for something different, without getting into the actual place value, (tenths, hundredths etc.).

DIVISION 5

Division with extended decimals

15.2 ÷ 5

```
  0 3 . 0 4
 ___________
5)1 ¹5 . 2 ²0
```

What to say:

Use a similar technique to that in Division 4, but at the point where there is a remainder of 2 in the tenths column, say:

Add a further zero after the 15.2, making 15.20. The 2 of these will make 20 of these.

Now continue dividing.

Is 21 ÷ 4 equal to $5\,^1/_4$, 5 r 1 or 5.25? Actually it is all of these. Which answer is the most suitable will depend on the context of the calculation.

Share 21 bars of chocolate between 4 people: 21 ÷ 4 = $5\,^1/_4$ bars each.
Share 21 pence between 4 people: 21p ÷ 4 = 5p r 1p
Share £21 between 4 people: £21 ÷ 4 = £5.25

And how about this one?
21 people want to go on a fairground ride. Each ride car takes 4 people. How many cars are needed?
This situation involves division of 21 by 4, but the answer is none of the above. It will require 6 cars.

DIVISION 6

Division with recurring decimals

29 ÷ 6

$$\begin{array}{r} 0\ \ 4\ .\ 8\ \ 3\ \ 3\ \ 3 \\ 6\overline{)2\ {}^{2}9\ .\ {}^{5}0\ {}^{2}0\ {}^{2}0\ {}^{2}0} \end{array}$$

$29 \div 6 = 4.8\dot{3}$

("4.83 recurring")

What to say:
Use a similar technique to that in Division 4, but at the point where there is a remainder of 5 units, add a point and a zero, making 29.0 (which has the same value as 29).
Continue adding zeros and dividing further. If a repeating (recurring) pattern emerges, write the answer as a recurring decimal.

One figure 3 with a dot above is the minimum notation required to show the fact that the 3 recurs. More can be shown if required, e.g. $4.833\dot{3}$.

If more than one figure recurs, dots should be placed over all the recurring figures (or just the first and last if it is a long sequence).

$15 \div 7 = 2.\dot{1}\dot{4}\dot{2}\dot{8}\dot{5}\dot{7}$ or $15 \div 7 = 2.\dot{1}4285\dot{7}$

DIVISION 7

Division by decimal numbers

4.71 ÷ 0.3

x 10 ↓ x 10 ↓

47.1 ÷ 3

$$3\overline{)4\,{}^{1}7\,.{}^{2}1}$$
$$\text{answer: } 1\;5\,.\,7$$

What to say:

4.71 divided by 0.3 will have the same answer as 47.1 divided by 3. Therefore, avoid dividing by a decimal by scaling up both numbers until the number you are dividing by (the divisor) is a whole number.

Children may question why the answer may larger than the starting number, as concept of sharing creates the idea that something is divided in to parts which are smaller than the original whole. Essentially, the answer to a division is larger than the starting number when the divisor is less than 1. Reminding children that division ***can*** be viewed as repeated subtraction (see Division 2) helps explain this:

Imagine 15 ÷ 0.5 as "How many times can you take away a half from 15?" Because the half is less than 1, it can be done more than 15 times (30 times, in fact).

CMS TRIAL PLAN FOR WHEN TO TEACH STRATEGIES, AND TO WHOM

	Top third				Middle third				Lower third			
Year 5	A5 to 1000 inc money	S4 to 1000 inc money	M2x to 100	D2x to 100	A5 to 1000 inc money	S3 to 1000	M2x to 100	D2x to 100	A1,2,3 as required	S1,2 as required	M1x to 100	D1x to 100
			M3 to TUxTU	D4 to 2 d.p.		S4 to 1000 inc money	M2 as required	D3 to HTU ÷ U inc money	A4 to 100/1000	S3 to 100	M1,2 as required	D1,2 as required
			M4 to TU x TU				M3 to TU x U		A5 to 100/1000		M3 to TUxU	D3 to TU ÷ U inc money
			M5 to HTU x U				M4 to TU x U inc money				M4 to TUxU inc money	
Year 6	A6 to 10000 and 2dp	S5 to 1000	M3x to 1000	D3x to 1000 inc 2 d.p.	A6 to 1000 and 2dp	S4 to 1000 inc money	M3x to 100	D3x to 100 inc 2 d.p.	A1.2.3 as required	S1,2 as required	M2x to 100	D2x to 100
			M6 to HTU x U	D5 to 2 d.p.		S5 to 1000	M4 to TU x TU	D4 to 2 d.p.	A4 to 100 and money	S3 to 1000	M1,2 as required	D1,2 as required
			M7 to TU.t x U				M5 to HTU x U inc money		A5 to 100 and money	S4 to 1000 inc money	M4 to HTUxU	D3 to HTU ÷ U inc money
							M6 to HTU x U				M5 to TUxU inc money	
							M7 to TU.t x U					
Year 7	A6 to 1000000 and 3dp	S5 to 10000 and 2dp	M3x to 10000	D3x to 10000 inc 3 d.p.	A6 to 10000 and 3dp	S5 to 1000 and 2dp	M3x to 1000	D3x to 1000 inc 2 d.p.	A1,2,3,4 as required	S1,2,3 as required	M3x to 100	D3x to 100 inc 1 d.p.
			M7 to HTU.t x U.t	D5, D6, D7			M7 to TU.t x U.t	D5, D6	A6 to 100 and 2dp	S5 to 1000 and 1 d.p	M1,2 as required	D4 to 2 d.p.
											M6 to HTUxU inc money	
Year 8	A6 unlimited	S5 to 1000000 and 3dp	M3x unlimited	D3x unlimited	A6 to 1000000 and 3dp	S5 to 10000 and 3dp	M3x to 10000	D3x to 10000 inc 2 d.p.	A1,2,3,4 as required	S5 to 1000 and 2dp	M3x to 1000	D3x to 1000 inc 2 d.p.
			M7 unlimited	D5, D6, D7			M7 to HTU.th x U.t	D5, D6, D7	A6 to 1000 and 2dp		M7 to HTU.t x U	D5 to 2 d.p.